MAGNETIZATION DYNAMICS COUPLED WITH SPIN & SPIN WAVES

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- **Several examples for self-consistent calculation of spin transport and magnetization dynamics**

Hyun-Woo Lee (POSTECH)

→ Tips about how to implement Heat Transport

→ Issue: Coarse graining to properly consider T and grad T in Micromagnetics

- **Domain wall motion induced by propagating spin waves**

Hiroshi Kohno (Osaka Univ.), **Soo-Man Seo** (KU)

Magnetization Dynamics + Diffusive Spin Transport

Ex1. Current-induced excitation of single FM

In multilayered structure, the 2nd ferromagnet (= polarizer) is not essential for current-induced magnetic excitation when the magnetization is laterally inhomogeneous

→ Lateral spin diffusion

→ Theory: Polianski and Brouwer, PRL 92, 026602 (2004)

→ Experiment: Ozyilmaz et al. PRL 93, 176604 (2004)

Not applicable to describe the magnetic excitation in a **single** ferromagnet

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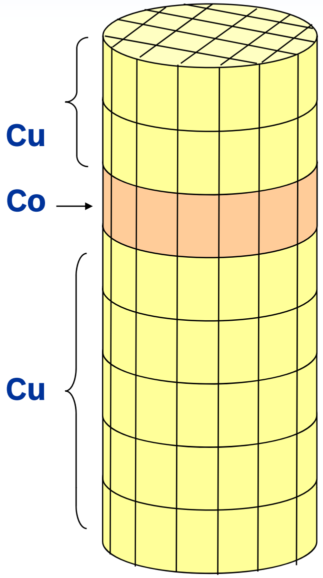
Magnetization Dynamics + Diffusive Spin Transport

Ex1. Current-induced excitation of single FM

→ Self-consistent calculation of LLG and diffusive spin transport equation for full 3D structure including leads:

$$\begin{aligned}\partial_t \mathbf{m} = & - \gamma_F(\mathbf{m}\times\mathbf{H}_{eff}) + \alpha\mathbf{m}\times\partial_t\mathbf{m} \\ & + \gamma_F/(M_s t_F)\left[\mathbf{J}_s|_{-t_F/2} - \mathbf{J}_s|_{+t_F/2}\right], \\ \partial_t\boldsymbol{\mu}_s & + \nabla\cdot\mathbf{J}_s = -\gamma_N(\boldsymbol{\mu}_s\times\mathbf{H}_{ext}) - \boldsymbol{\mu}_s/\tau_{sf}.\end{aligned}$$

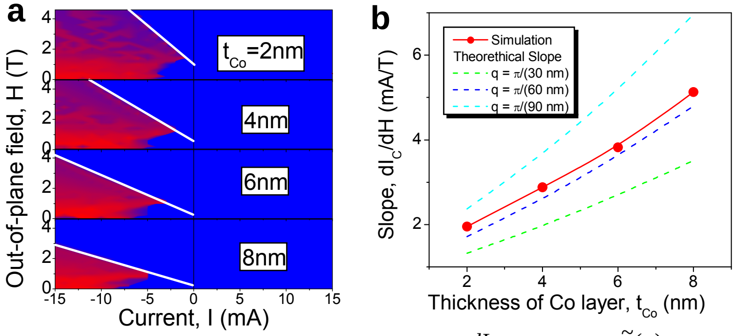
$$\begin{aligned}J_e &= (G_\uparrow + G_\downarrow)\Delta\mu_e/e + (G_\uparrow - G_\downarrow)\mathbf{m}\cdot(\boldsymbol{\Delta\mu_s}/e) \\ \mathbf{J}_s &= (\hbar/2e^2)[Re(G_{\uparrow\downarrow})\mathbf{m}\times(\mathbf{m}\times 2\boldsymbol{\Delta\mu_s} \pm \hbar\partial_t\mathbf{m}), \\ &- ((G_\uparrow + G_\downarrow)\mathbf{m}\cdot\boldsymbol{\Delta\mu_s} - (G_\uparrow - G_\downarrow)\Delta\mu_e)\,\mathbf{m}],\end{aligned}$$



Magnetization Dynamics + Diffusive Spin Transport

Ex1. Current-induced excitation of single FM

Magnetic cell size = 60 x 30 x t_{Co} nm^3



$$\frac{dI_C}{dH} = \frac{e}{\hbar} S M_S t_{Co} \frac{\tilde{\alpha}(q)}{S_1(q)}.$$

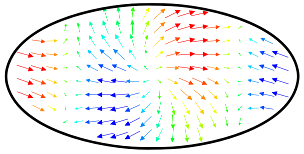
$\tilde{\alpha}(q)$: Renormalized damping

$S_1(q)$: STT magnitude

Magnetization Dynamics + Diffusive Spin Transport

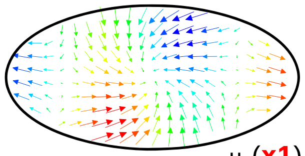
Ex1. Current-induced excitation of single FM

M

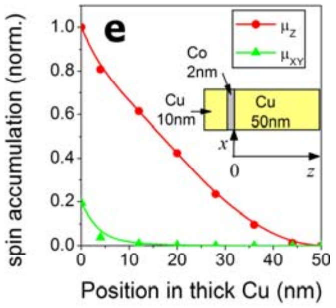


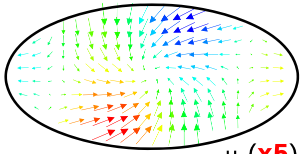
μ

I > Ic

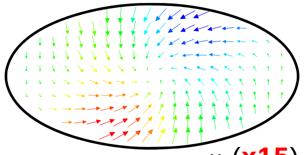


μ (**x1**) at z = 0nm





μ (**x5**) at z = 6nm



μ (**x15**) at z = 18nm

Magnetization Dynamics + Diffusive Spin Transport

Ex2. Charge and spin currents caused by spin motive force

- Spatiotemporal change of magnetization

➔ Spin Motive Force (SMF) [1] $E_{Si}^{\uparrow\downarrow} = \pm\frac{\hbar}{2e}(\partial_i \mathbf{m} \times \partial_t \mathbf{m}) \cdot \mathbf{m}$

Theories

• Volovik, J. Phys. C. ’87 / Barnes & Maekawa, PRL ’07 / Saslow, PRB ’07
• Ohe et al, PRL ’07 / Tserkovnyak et al. PRB ’07-’10 / Duine, PRB ’08 / Zhang PRL ‘09

spin current $\mathbf{j}_i^s = \frac{g\mu_B}{2e}(G^\uparrow E_i^\uparrow - G^\downarrow E_i^\downarrow)\mathbf{m} = \frac{g\mu_B \hbar G_0}{4e^2}(\partial_t \mathbf{m} \times \partial_i \mathbf{m})$

➔ Spin current that causes spatial dependent enhancement of Gilbert damping (similar to spin pumping to normal metal in contact: Tserkovnyak et al. PRL ‘02)

$$\partial_t \mathbf{m} = -\gamma \mathbf{m} \times \mathbf{H}_{\mathrm{eff}} + \mathbf{m} \times (\mathcal{D} \cdot \partial_t \mathbf{m})$$

$$\mathcal{D}_{\alpha\beta} = \alpha_0 \delta_{\alpha\beta} + \eta \sum_i (\mathbf{m} \times \partial_i \mathbf{m})_\alpha (\mathbf{m} \times \partial_i \mathbf{m})_\beta \qquad \eta = \frac{g\mu_B \hbar}{4e^2 M_s} G_0 \sim 0.5 nm^2 \;\; for\ Py$$

- For DW width = 5 nm

➔ additional damping = 0.5 nm^2/(5 nm)^2 = 0.02

Magnetization Dynamics + Diffusive Spin Transport

Ex2. Charge and spin currents caused by spin motive force

$$\frac{\partial \mathbf{m}}{\partial t} + \nabla \cdot J = -\frac{1}{\tau_{ex} M_s} \mathbf{m} \times \mathbf{M}(x,t) - \langle \mathbf{\Gamma}(\mathbf{s}) \rangle$$

⇩ Take longitudinal component

$$\frac{\partial n_s}{\partial t} + \hat{\mathbf{m}} \cdot \langle \mathbf{\Gamma}(\mathbf{s}) \rangle = \frac{1}{e}\frac{\partial j_s}{\partial x} \qquad \frac{1}{2}\hat{\mathbf{m}} \cdot \langle \mathbf{\Gamma}(\mathbf{s}) \rangle = \frac{n^\uparrow}{\tau^\uparrow} - \frac{n^\downarrow}{\tau^\downarrow}$$

$$\boxed{\begin{aligned} \frac{\partial n^\uparrow}{\partial t} \qquad\qquad &= \frac{\sigma^\uparrow}{e}\frac{\partial E_s^\uparrow}{\partial x} \\ \frac{\partial n^\downarrow}{\partial t} \qquad\qquad &= \frac{\sigma^\downarrow}{e}\frac{\partial E_s^\downarrow}{\partial x} \end{aligned}}$$

n = electron density
D = diffusion const.
τ_{sf} = spin relaxation time
E_s = electric field due to SMF

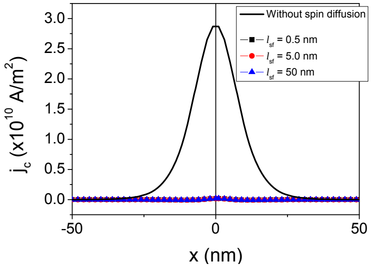
Magnetization Dynamics + Diffusive Spin Transport

Ex2. Charge and spin currents caused by spin motive force

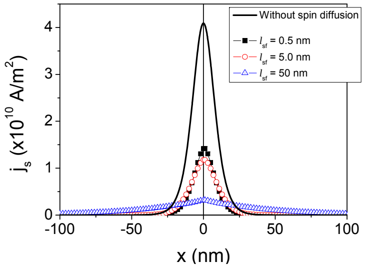
[Model system]

• 1D domain wall (DW) oscillator ➔ fixed position ($dX_{DW}/dt = 0$), but rotating ($d\phi/dt \neq 0$) & DW width = 10 nm, ω = 10 GHz, λ_{sf} = 0.5, 5, 50 nm

[Charge current, j_c]



[Spin current, j_s]



• When spin diffusion is turned on, j_c almost vanishes and j_s significantly reduces.
• The reduction in j_s depends on the spin diffusion length.

Magnetization Dynamics + Ballistic Spin Transport
Ex3. Nonlocal STT in a very narrow domain wall

- Wide DW

- Narrow DW

• The β-term must be non-zero and $\propto \exp(1/\lambda_{DW})$ where λ_{DW} is the DW width

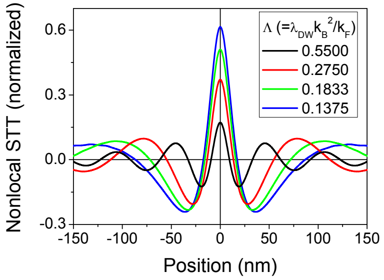
[Xiao et al. PRB **73**, 054428 (2006)]

[Tatara, …, KJL, JPSJ **76**, 054707 (2007)].

• Narrow wall ➔ good for the high density storage

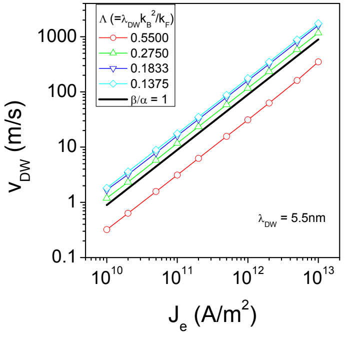
Magnetization Dynamics + Ballistic Spin Transport
Ex3. Nonlocal STT in a very narrow domain wall

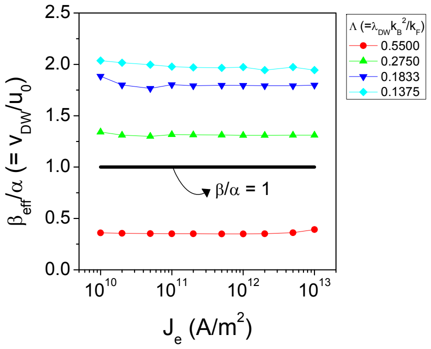
- We *self-consistently* solve the LLG and the semi-classical spin transport equation.
- Model system = Perpendicular materials
  - Ku = 3.3x10^6 erg/cm^3, Ms = 650 emu/cm^3, A = 2.0x10^{-6} erg/cm, α = 0.1,W = 100 nm & t = 10 nm, β_{spin_relax}=0
  - Variable: $\Lambda \equiv \lambda_{DW} k_B^2 / k_F \quad E_F = \hbar^2 k_F^2 / 2m \quad \& \quad E_{ex} = \hbar^2 k_B^2 / m$



Magnetization Dynamics + Ballistic Spin Transport
Ex3. Nonlocal STT in a very narrow domain wall

u_0 = spin current velocity





- $v_{DW} \propto (J_e)^1$
- v_{DW}/u_0 = constant

➔ $\beta_{nonlocal}$ acts like β_{spin_relax}.

Numerical approach for spin caloritronics?

- **Coupled dynamics of MAGNETIZATION + SPIN ➔ Well established**
- **MAGNETIZATION + SPIN + HEAT (Temperature)**
  - **(Static) Spin-dependent thermoelectrics (van Wees group's talk)**
  - **Magnon-driven Spin-Seebeck (Ohe et al. PRB '11)**
  - **Thermal STT + M dynamics (self-consistent)**
  - **Phonon-driven Spin-Seebeck**
  - **…**
- **An important issue**
  - **How to properly consider temperature and its gradient?**

Coarse graining in Micromagnetics

➔ **Spin waves with a shorter wavelength than block size is neglected**

➔ **Renormalization of Exchange Constant A [Grinstein and Koch, PRL ’03]**

$$T(l) = T_c/[1 + e^{\epsilon l}(T_c/T(0) - 1)]$$

Tc : Curie temperature
ε = d-2
l = ln(D/a)
where D is the unit cell size
and a is the lattice parameter

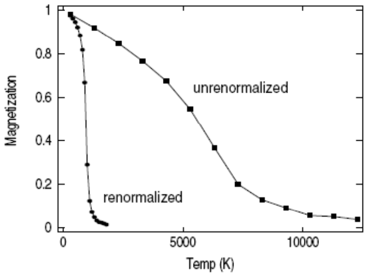
Renormalization of exchange constant A

$$A(b) = (\tilde{T}/b\tilde{T}_c) \times [1 + b(\tilde{T}_c/\tilde{T} - 1)]A$$

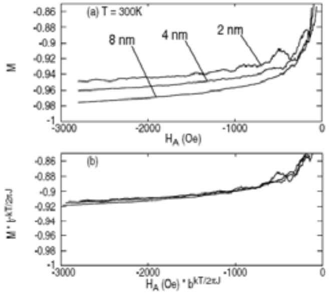
A(b=0) = 1.3x10^{-6} erg/cm
➔ For 3 nm unit cell, A(l) ~ 0.9x10^{-6} erg/cm

Coarse graining in Micromagnetics

Curie temperature correction!



Renormalization of M and H



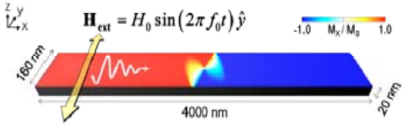
Renormalization of Other Parameters?

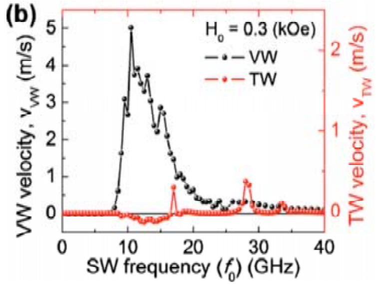
➔ **As a first step, renormalization of anisotropy and spin torque is in progress (collaboration with Prof. H. Kohno).**

Do you want to move domain wall (DW))?

➔ **Magnetic field / Current / Heat flow**

➔ **Spin waves (SW):** Han et al. APL 94, 112502 (2009): First modeling study



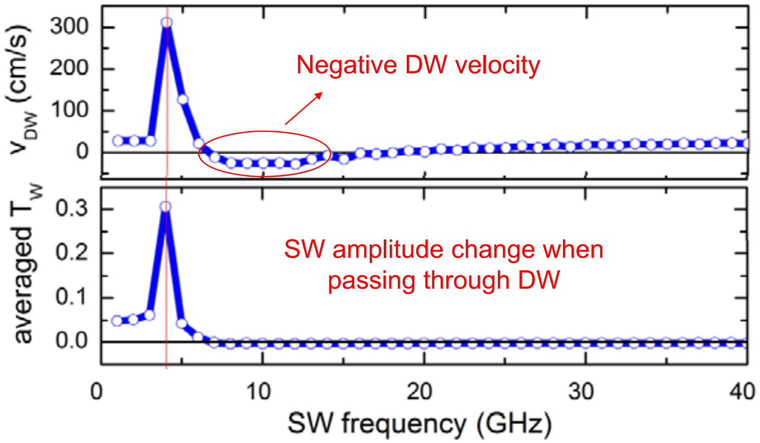


➔ **Jamali, Yang, KJL, APL 96, 242501 (2010): SW can assist current-induced DW motion**

➔ **Seo, Lee, Kohno, KJL, APL 98, 012514 (2011): Vortex DW is faster than Transverse DW**

Numerical results on SW-induced DW motion

1D Neel DW



- v_{DW} > 0 (DW moves along SW propagation direction) or v_{DW} < 0 depending on SW frequency
- Amplitude change + ?

Understanding SW-induced DW motion

(Prof. H. Kohno, kohno@mp.es.osaka-u.ac.jp)

- We introduced SW spin current and SW momentum current
- SW spin current ($\boldsymbol{J}_\mathrm{s}$)

$$\frac{\partial M}{\partial t} = \gamma H_\mathrm{eff} \times M + \frac{\alpha}{M_\mathrm{s}} M \times \frac{\partial M}{\partial t}$$

$$\dot{M} + \mathrm{div} J_\mathrm{s} = T. \longrightarrow \boxed{J_\mathrm{s} = \gamma A M \times \nabla M,}$$

$$T = \gamma H' \times M + \frac{\alpha}{M_\mathrm{s}} M \times \dot{M}$$

- SW momentum current ($\boldsymbol{J}_\mathrm{m}$)

Lagrangian Density $\mathcal{L} = -(M_\mathrm{s}/\gamma)\,\dot{\phi}\,(\cos\theta + 1) - \frac{1}{2}A\,(\nabla M)^2 - V(M)$

Energy-Momentum Tensor $T_{\mu\nu} = (\partial_\mu q)\,\partial\mathcal{L}/\partial(\partial_\nu q) - \delta_{\mu\nu}\mathcal{L}$

Momentum Density $T_{ij} = -A\,(\partial_i M \cdot \partial_j M) - \delta_{ij}\mathcal{L}$

SW momentum current $\boxed{\boldsymbol{J}_\mathrm{m} = \frac{1}{2} A\left[(\partial_x M)^2 - (\partial_y M)^2\right] - V.}$

Understanding SW-induced DW motion

(Prof. H. Kohno)

- SW spin current ($\boldsymbol{J}_\mathrm{s}$)

$$\pm\frac{\dot{X}}{\lambda} + \Delta j_\mathrm{s} = \frac{K_\perp S}{2\hbar}\sin 2\phi_0 + \alpha\dot{\phi}_0, \qquad \Delta j_\mathrm{s} = \pm 2\,(JS/\hbar)\,|u_x u_y|\,q.$$

  - $\boldsymbol{J}_\mathrm{s}$ ➔ negative DW velocity / proportional to wavevector *q*, exchange *A*, and SW amplitude *u*.

- SW momentum current ($\boldsymbol{J}_\mathrm{m}$) $\pm 2 s_0\,\dot{\phi}_0 - \Delta j_\mathrm{m} = -\alpha s_0 \frac{2\dot{X}}{\lambda}$

$$\Delta j_\mathrm{m} = \frac{S^2}{4Ka^3}\left\{(Jq^2)^2 - K(K + K_\perp)\right\}\left\{u^2|_{x=\infty} - u^2|_{x=-\infty}\right\}.$$

  - $\boldsymbol{J}_\mathrm{m}$ ➔ DW velocity changes its sign depending on SW frequency / proportional to Amplitude change.

- Analysis based on these two SW currents are in progress.

Summary

1. **Some examples for self-consistent of spin transport and magnetization dynamics**
2. **When including Heat, coarse graining should be done.**
3. **SW can move DW: DW velocity versus SW frequency would be understood based on SW spin and momentum currents.**