Superconducting proximity effects in magnetic metals

E. A. Demler
Department of Physics, Stanford University, Stanford, California 94305

G. B. Arnold* and M. R. Beasley
Department of Applied Physics, Stanford University, Stanford, California 94305
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We explain the basic physics behind oscillatory effects in superconductor/metallic ferromagnet (S/F) sandwiches, and describe the important effects of the spin orbit scattering in these systems. We find that spin-orbit scattering plays a major role in the physics of the superconducting proximity effect with a conducting ferromagnet. As examples, we present calculations of the T_c of an S/F bilayer and the Josephson current (near T_c) of an $S/F/S$ trilayer. [S0163-1829(97)06521-1]

I. INTRODUCTION

What happens when a ferromagnetic layer (F) is placed in contact with a superconductor (S)? The purpose of this work is to consider the answer to this question for the specific case in which the ferromagnet is a good conductor and the superconductor is an s-wave superconductor. We also restrict our consideration to the most relevant case when the Curie temperature of the ferromagnet is much greater than the superconducting T_c.

This situation has been treated by several authors, most recently by Buzdin *et al.*,[1–5] who have observed that the exponentially decaying Cooper pair density in the ferromagnet also has an oscillatory character, indicating that the Cooper pair acquires a spatially dependent phase in the ferromagnetic layer. This causes an exchange field dependent oscillation in the critical current of SFS sandwiches, and in the T_c of SF bilayers and multilayers. When the Josephson coupling energy is negative, one has a so-called π junction, for which the minimum energy configuration corresponds to a phase shift of π in the macroscopic phase difference across the junction. Despite several experimental studies,[6] there is no definitive experimental evidence for these predictions.

The purpose of this paper is to review these earlier calculations, so as to reveal more clearly the underlying physics, and to extend them so as to include a more general treatment of the important effects of the spin-orbit scattering. As we shall see, spin-orbit scattering plays a major role in the physics of the proximity effect with a ferromagnet.[7,12] We do not consider here a specific example of the experimental situation, which is unclear and controversial. We simply note that spin-orbit scattering is relevant in conductors containing large Z elements. An assessment of the current experimental situation in light of our results will be presented in a subsequent paper.

Before turning to the detailed microscopic theory of these effects, it is well to review the basic physics behind them, and to present simple physical arguments which justify them. The fundamental feature to be justified is the oscillating pair density.

For simplicity, we first consider the situation in which spin is a good quantum number (i.e., there is no spin-orbit interaction). Imagine a Cooper pair being adiabatically transported across an SF interface with its electron momenta aligned with the interface normal. Upon entering the F region, where the pair is not an eigenstate, it becomes an evanescent state, decaying exponentially on the length scale ξ_0, the normal metal coherence length. In addition, the up spin electron in the pair lowers its potential energy by h, the exchange field energy in the ferromagnet, while the down spin electron raises its potential energy by the same amount. In order for each electron to conserve its total energy, the up spin electron must increase its kinetic energy, while the down spin electron must decrease its kinetic energy, to make up for these additional potential energies in F.

So for a pair, shown on top of Fig. 1, entering into a ferromagnetic region results in acquiring a center of mass momentum $Q=2h/v_F$. The fermionic antisymmetry requires us to consider the pair described above together with the pair which has the down spin and up spin electrons in-

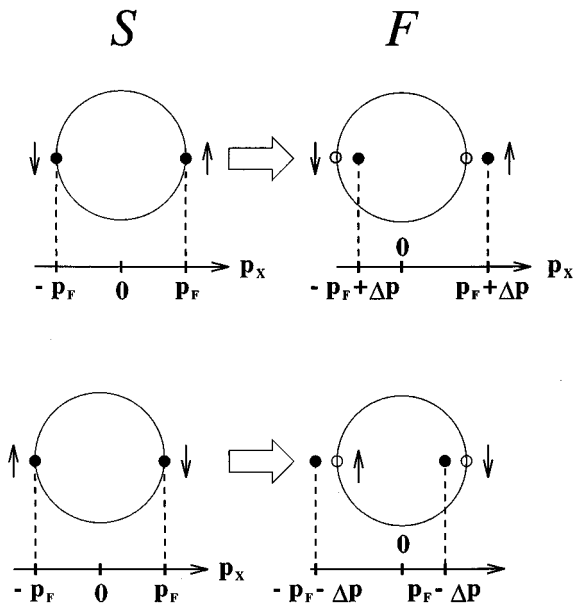

FIG. 1. Coopper pair in superconductor and ferromagnet. $\Delta p = h/v_F$.

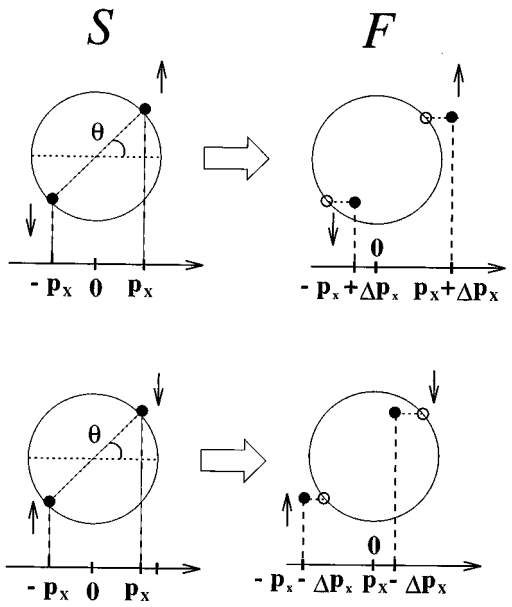

FIG. 2. Cooper pair in superconductor and ferromagnet for a general direction of the relative momentum of two electrons. $\Delta p_x = h/v_F \cos\theta$.

terchanged in momentum space (shown on the bottom of Fig. 1). The latter gains a center of mass momentum $-Q$ upon crossing the SF boundary. Combining the two pairs into a singlet combination we see that the overall effect of the exchange field in the F region on a singlet Cooper pair is to give it a spatial modulation. Hence if the wave function of the pair in a superconductor is $\Phi(x_1-x_2)$, where x_1 and x_2 are the coordinates of the two electrons, in a ferromagnet the wave function becomes $\cos[Q(x_1+x_2)]\Phi(x_1-x_2)$.

In the more general case when the electrons in a pair have their momenta at an angle θ with respect to the interface normal (see Fig. 2), the additional momentum that each electron gains after crossing the SF boundary is $\Delta p_x = h/v_F \cos\theta$ and $\Delta p_y = \Delta p_z = 0$. Here we used the fact that momentum is conserved in the direction parallel to the interface to reason that the electrons may change their momenta in the x direction only. The modulation factor of the pair shown in Fig. 2 in the F region is $\cos[h(x_1+x_2)/v_F \cos\theta]$.

The overall Cooper pair distribution is then obtained by accounting for all possible angles of incidence for the pair, so it is proportional to

$$\int_0^1 d(\cos\theta)\cos x = \int_0^1 d(\cos\theta)\cos\left(\frac{2hx}{v_F\cos\theta}\right) \approx \frac{\sin(x/\xi_{m0})}{(x/\xi_{m0})}. \tag{1}$$

[We assume, for simplicity, that $v_F/(2h) = \xi_{m0} \ll \xi_0$, so that the overall exponential decay of the Cooper pair in F over ξ_0 may be neglected.] Thus the Cooper pair distribution oscillates on the scale set by the length ξ_{m0}. This establishes simply the physical origin of the oscillations.

The physical picture of the proximity effect in a clean ferromagnetic conductor is therefore very similar to the Fulde-Ferrel-Larkin-Ovchinikov (FFLO) effect.[8,9] In the FFLO state a superconducting order parameter is generated in the presence of an exchange field, and it turns out that energetically a spherically symmetric distribution of the electrons is less favorable than the distribution extended along one of the directions perpendicular to the exchange field. So Cooper pairs with shifted center of mass momenta appear, and an inhomogeneous distribution function, similar to the one we described earlier, develops.

We now want to understand the effect of elastic potential (nonmagnetic and non-spin-orbit) scattering. As usual we only need to consider the processes in which the two electrons in a Cooper pair are scattered by the same impurity into states with opposite momenta, since all the other scattering events are incoherent in the pairing process. An interesting question now is what happens to the center of mass momentum that the Cooper pairs acquire upon entering the ferromagnet [and ultimately the modulation factor $\cos[h/(v_F \cos\theta)(x_1+x_2)]$ for singlet pairs). In a clean limit we found that this center of mass momentum was a function of the relative momentum of the two electrons and remained constant throughout the whole trajectory of the pair in the F region. Now, as a result of the multiple scattering processes, the center of mass momentum will vary along the trajectory of the pair together with the relative momentum. It is important to realize, however, that the center of mass momenta that we need to consider are always in the x direction, because the scattering events leading to the other directions are incoherent. And since scattering on impurities does not change the energy of the electrons, we can again use the energy argument introduced above to deduce that in the dirty limit we have the same local relationship between the direction of the relative momentum of the electrons and the magnitude of the center of mass momentum $Q = 2h/v_F \cos\theta$ as in the clean limit. This observation allows us to treat the case of isotropic and strong potential scattering (l, the mean free path due to scattering, is much smaller than any other length in the problem) similarly to the clean case, only instead of integrating over the angles of incidence of the pairs we integrate over all possible intermediate orientations. The only thing that we will need to add for the case of impurities, present in a ferromagnet, is an extra decay of the Cooper pairs due to scattering. This decay has the important feature that it depends on the direction of the momenta of the electrons in the Cooper pair and, in the dirty limit, the net scattering rate for the pairs is proportional to $\cos\theta/\tau$, yielding an effective mean free path for pairs of $l/\cos\theta$.[10] The pair distribution function, accounting for this scattering effect, is thus proportional to

$$\int_0^1 d\,\cos\theta e^{-(x/l)\cos\theta}\cos\left(\frac{2hx}{v_F\cos\theta}\right) \tag{2}$$

which integrates approximately, yielding a Cooper pair density proportional to

$$\mathrm{Re}\left(\frac{e^{-(1-i)x/\xi_m + 3\pi i/8}}{\sqrt{x\xi_m/l^2}}\right),$$

where $\xi_m = \sqrt{l\xi_{m0}/2}$.

In the integral (1) the rapid oscillations of the integrand for small values of $\cos\theta$ ensure that the most important pair trajectories contributing to this effect are those which are nearly perpendicular to the SF interface. Formula (2) shows

that if there is strong elastic scattering in F, however, the values of $\cos\theta$ less than one can contribute as well, because of scattering into the low θ direction. The average $\cos\theta$ value contributing dominantly then decreases. This lowers the effective period of the oscillations, as well as introducing decay of the oscillations due to the fact that momentum parallel to the interface is no longer conserved within the F layer. The oscillations are now damped on the same length scale at which they oscillate. The effect of the scattering is to average over the *effective magnitude* of the exchange field from a minimum value equal to $h(\cos\theta=1)$, to a maximum which approaches infinity ($\cos\theta=0$), resulting in a shorter oscillation period.

In the presence of spin-orbit scattering, the spin singlet Cooper pair wave function decays (to a spin triplet) by spin-flip scattering, and the $+Q$ momentum pairs mix with the $-Q$ momentum (spin-exchanged) pairs. The decay to a triplet is a pair breaking effect, giving the pairs a finite lifetime because the intermediate state in spin-orbit scattering is a triplet whose energy depends upon the exchange field. Another effect of the spin orbit scattering is to mix the Copper pair with its spin-exchanged counterpartner. This causes a pair to ''see'' an exchange field which changes sign at a rate proportional to $1/\tau_{so}$, decreasing the average h field experienced by the pair, hence increasing the period of the oscillations.

For very strong spin-orbit scattering, the Cooper pair can no longer be regarded as a spin singlet for any reasonable length of time, in particular for the time $\Delta t=1/h$, when $\tau_{so}<1/h$, so that the oscillation effect disappears. Put another way, the electrons which make up the pair can no longer be regarded as eigenstates of spin in this case.

For pedagogical purposes, we ignore potential scattering for the moment, and consider the effects of spin-orbit scattering alone. In actuality, spin-orbit scattering is inevitably accompanied by potential scattering. The additional influence of potential scattering in the dirty limit will be accounted for subsequently.

The decay and mixing processes change the energy of the pairs, and introduce a lifetime effect through equations for the forward (f_Q) and backward, spin-exchanged (f_{-Q}) portions of the Cooper pair wave function (see Sec. III of this paper):

$$[E-(2h+2i/\tau_{so})]f_Q+2i/\tau_{so}f_{-Q}=0,$$

$$[E-(-2h+2i/\tau_{so})]f_{-Q}+2i/\tau_{so}f_Q=0.$$

(In the absence of the exchange field h, one sees immediately that $f_Q=f_{-Q}$ and the spin orbit scattering has no effect on the pairs.) Solving for E yields a (complex) pair energy

$$E=2i/\tau_{so}-\sqrt{(2h)^2-(2/\tau_{so})^2}=v_F/\zeta_{m0}$$

and hence a complex momentum

$$Q=E/(v_F\cos\theta)=1/(\zeta_{m0}\cos\theta).$$

Accounting for both directions of momentum, we get a Cooper pair density proportional to

$$\int_0^1 d(\cos\theta)\,\mathrm{Re}(e^{ix/(\zeta_{m0}\cos\theta)}).$$

This integrates approximately to

$$\mathrm{Re}\left(\frac{e^{ix/\zeta_{m0}}}{ix/\zeta_{m0}}\right).$$

One can see that the existence of oscillations requires $h>1/\tau_{so}$.

If we include strong elastic scattering as before, then this becomes, approximately

$$\mathrm{Re}\left(\frac{e^{-(1-i)(x/\zeta_m)+3\pi i/8}}{\sqrt{x\zeta_m/l^2}}\right),$$

where $\zeta_m=\sqrt{l\zeta_{m0}/2}$.

II. EILENBERGER EQUATIONS IN THE PRESENCE OF THE EXCHANGE FIELD AND THE SPIN ORBIT SCATTERING

In this section we briefly review the derivation of the Eilenberger equations and show how they can be generalized to account for the presence of an exchange field and spin orbit scattering.

We perform all the calculations in the Matsubara imaginary time formalism and our four-coordinate x stands for $(\tau,\mathbf{r})$. Following Maki[11] we introduce a spinor representation of the fermion operators:

$$\Psi(x)=\begin{pmatrix}\psi_\uparrow(r,\tau)\\ \psi_\downarrow(r,\tau)\\ \psi_\uparrow^\dagger(r,\tau)\\ \psi_\downarrow^\dagger(r,\tau)\end{pmatrix},\quad \Psi^\dagger(x)=(\psi_\uparrow^\dagger(r,\tau)\psi_\downarrow^\dagger(r,\tau)\psi_\uparrow(r,\tau)\psi_\downarrow(r,\tau)). \tag{3}$$

In the absence of the impurities the Gor'kov equations for a 4×4 matrix Green's function $\hat{G}(x_1,x_2)=-\langle T_\tau\Psi(x_1)\Psi^\dagger(x_2)\rangle$ can be written in two forms, corresponding to writing the equations of motion of the particles at x_1 or x_2,

$$\frac{\partial\hat{G}(x_1,x_2)}{\partial\tau_1}=\left\{\left(\frac{\hbar^2}{2m}\nabla_1^2+\mu\right)\hat{\rho}_3+\hat{\Delta}(r_1)+h(r_1)\hat{\rho}_3\hat{\sigma}_3\right\}\hat{G}(x_1,x_2)-\hat{1}\delta^3(r_1-r_2)\delta(\tau_1-\tau_2),$$

$$-\frac{\partial\hat{G}(x_1,x_2)}{\partial\tau_2}=\hat{G}(x_1,x_2)\left\{\left(\frac{\hbar^2}{2m}\nabla_2^2+\mu\right)\hat{\rho}_3+\hat{\Delta}(r_2)+h(r_2)\hat{\rho}_3\hat{\sigma}_3\right\}-\hat{1}\delta^3(r_1-r_2)\delta(\tau_1-\tau_2), \tag{4}$$

where

$$\hat{\Delta}(r)=i\hat{\rho}_+\hat{\sigma}_2\Delta(r)-i\hat{\rho}_-\hat{\sigma}_2\Delta^*(r),$$

$$\Delta(r)=\lambda\langle\psi_\downarrow(r)\psi_\uparrow(r)\rangle=\frac{i\lambda}{2}\,\mathrm{Tr}[\hat{\rho}_+\hat{\sigma}_2\hat{G}(x,x)]. \tag{5}$$

$\hat{\rho}$ and $\hat{\sigma}$ are the Pauli matrices in the particle-hole and spin spaces correspondingly and λ is the BCS coupling constant.

The Gor'kov equations carry the information about both the macroscopic fields and the excitation spectrum. In particular, it is the the center of mass spatial dependence that gives the macroscopic fields and the relative coordinate dependence that gives the excitation spectrum. Since we are not interested in the excitation spectrum but only in the macroscopic pair wave function, we integrate over the relative coordinates of two particles sacrificing our knowledge of the excitation spectrum for the sake of getting simpler equations.

We want to separate the center of mass and relative motions inside the Green's functions. So, from r_1 and r_2 we go to $R=(r_1+r_2)/2$, the position of the center of mass, and $r=r_2-r_1$, the relative coordinate of the two particles. We also make a Fourier transform in the imaginary time domain using the fact that the time homogeneity is not broken and everything depends on $\tau_2-\tau_1$ only,

$$\left[\frac{\hbar^2}{2m}\left(\frac{1}{2}\frac{\partial}{\partial R}-\frac{\partial}{\partial r}\right)^2+\mu\right]\hat{G}_\omega\left(R-\frac{r}{2},R+\frac{r}{2}\right)=\left\{-i\omega\hat{\rho}_3-\hat{\rho}_3\hat{\Delta}\left(R-\frac{r}{2}\right)-h\left(R-\frac{r}{2}\right)\hat{\sigma}_3\right\}\hat{G}_\omega\left(R-\frac{r}{2},R+\frac{r}{2}\right)+\hat{\rho}_3\delta^3(r),$$

$$\left[\frac{\hbar^2}{2m}\left(\frac{1}{2}\frac{\partial}{\partial R}+\frac{\partial}{\partial r}\right)^2+\mu\right]\hat{G}_\omega\left(R-\frac{r}{2},R+\frac{r}{2}\right)=\hat{G}_\omega\left(R-\frac{r}{2},R+\frac{r}{2}\right)\left\{-i\omega\hat{\rho}_3-\hat{\rho}_3\hat{\Delta}\left(R-\frac{r}{2}\right)-h\left(R-\frac{r}{2}\right)\hat{\sigma}_3\right\}+\hat{\rho}_3\delta^3(r). \tag{6}$$

We assume that the macroscopic fields h and Δ vary on the length scales bigger than the coherence length of material (S or F), and so we can replace the actual argument of the two functions $R\pm r/2$ by just R. Later we will be using the resulting equations when this condition is not rigorously satisfied. However, one can convince oneself that this procedure is a quasiclassical approximation and only results in averaging over the fast oscillations on the length scale of $1/k_F$.

We subtract the first equation of (6) from the second and get

$$\frac{\hbar^2}{m}\frac{\partial^2}{\partial R\partial r}\hat{G}_\omega\left(R-\frac{r}{2},R+\frac{r}{2}\right)=\{i\omega\hat{\rho}_3+\hat{\rho}_3\hat{\Delta}(R)+h(R)\hat{\sigma}_3\}\hat{G}_\omega\left(R-\frac{r}{2},R+\frac{r}{2}\right)-\hat{G}_\omega\left(R-\frac{r}{2},R+\frac{r}{2}\right)$$
$$\times\{i\omega\hat{\rho}_3+\hat{\rho}_3\hat{\Delta}(R)+h(R)\hat{\sigma}_3\}. \tag{7}$$

We can integrate over the energies of the relative motion of the two particles using two simple transformations

$$\hat{G}_\omega\left(R-\frac{r}{2},R+\frac{r}{2}\right)=\int\frac{d^3p}{(2\pi\hbar)^3}\,\hat{G}_\omega(R,\mathbf{p})e^{i\mathbf{p}\mathbf{r}/\hbar},$$

$$\hat{G}_\omega(R,\mathbf{n})=\int\frac{d\xi_p}{2\pi\hbar}\,\hat{G}_\omega(R,\mathbf{p}). \tag{8}$$

This gives

$$i\mathbf{v}\frac{\partial}{\partial\mathbf{R}}\hat{\mathbf{G}}_\omega(\mathbf{R},\mathbf{n})=\{i\omega\hat{\rho}_3+\hat{\rho}_3\hat{\Delta}(R)+h(R)\hat{\sigma}_3\}\hat{G}_\omega(R,\mathbf{n})-\hat{G}_\omega(R,\mathbf{n})\{i\omega\hat{\rho}_3+\hat{\rho}_3\hat{\Delta}(R)+h(R)\hat{\sigma}_3\}, \tag{9}$$

where $\mathbf{n}$ is a unit vector that carries information about the direction of the relative motion of two particles and $\mathbf{v}=\mathbf{p}/m\simeq v_F\mathbf{n}$.

At this point it is straightforward to introduce the effect of the impurities. It is simply a matter of insertion of the self-energy in Born approximation into (9),

$$i\mathbf{v}\frac{\partial}{\partial\mathbf{R}}\hat{\mathbf{G}}_\omega(\mathbf{R},\mathbf{n})=\{i\omega\hat{\rho}_3+\hat{\rho}_3\hat{\Delta}(R)+h(R)\hat{\sigma}_3-\hat{\rho}_3\Sigma_\omega(R,\mathbf{n})\}\hat{G}_\omega(R,\mathbf{n})-\hat{G}_\omega(R,\mathbf{n})\{i\omega\hat{\rho}_3+\hat{\rho}_3\hat{\Delta}(R)+h(R)\hat{\sigma}_3-\Sigma_\omega(R,\mathbf{n})\hat{\rho}_3\}, \tag{10}$$

where

$$\Sigma_\omega(R,\mathbf{n})=n(R)\int\frac{d\mathbf{n}'}{4\pi}\,\hat{U}(\mathbf{n}-\mathbf{n}')\hat{G}_\omega(R,\mathbf{n}')\hat{U}(\mathbf{n}'-\mathbf{n}),$$

$$\hat{U}(\mathbf{n}-\mathbf{n}')=U_1\rho_3+U_{so}i[\mathbf{n}\times\mathbf{n}']\alpha, \tag{11}$$

and α is an electronic spin operator $\alpha=[(1+\rho_3)/2]\sigma+[(1-\rho_3)/2]\sigma_2\sigma\sigma_2$.

Components of the matrix $\hat{G}$ are given by

$$\hat{G}_\omega(R,\mathbf{n})=\frac{mp_F}{2\pi^2\hbar^2}\begin{vmatrix} -\frac{i}{2}g_+ & 0 & 0 & \frac{-1}{2}f_+ \\ 0 & -\frac{i}{2}g_- & \frac{1}{2}f_- & 0 \\ 0 & \frac{1}{2}f^\dagger & \frac{i}{2}g_- & 0 \\ -\frac{1}{2}f_+^\dagger & 0 & 0 & \frac{1}{2}g_+ \end{vmatrix}_{(\omega,R,\mathbf{n})}. \tag{12}$$

When writing (12) we assumed purely singlet pairing, and we used the integral of the equation (10)

$$\mathbf{v}\frac{\partial}{\partial\mathbf{R}}[\mathrm{Tr}\,\hat{G}_\omega(R,\mathbf{n})]=0 \tag{13}$$

together with the asymptotic form of the Green's functions in the bulk of the superconductor to get the equality of the spin reversed Green's functions (so, that we have only two g's instead of four). And we also introduced numerical factors in order to have a simple assymptotic form of the f and g functions in the bulk of the superconductor: $f_\pm=\Delta/\sqrt{\omega^2+\Delta^2}$ and $g_\pm=\omega/\sqrt{\omega^2+\Delta^2}$ (for the case when $h=0$). Using the functions introduced in (12), Eqs. (10) and (11) become

$$\left(\widetilde{\omega}_\pm(r,v)\pm ih(r)-\frac{1}{2}\mathbf{v}\frac{\partial}{\partial\mathbf{r}}\right)f_\pm(\omega,r,v)=\widetilde{\Delta}_\pm(\omega,r,v)g_\pm(\omega,r,v),$$

$$\left(\widetilde{\omega}_\pm(r,v)\pm ih(r)+\frac{1}{2}\mathbf{v}\frac{\partial}{\partial\mathbf{r}}\right)f_\pm^\dagger(\omega,r,v)=\widetilde{\Delta}_\pm^*(\omega,r,v)g_\pm(\omega,r,v),$$

$$f_\pm^\dagger f_\pm+g_\pm^2=1,$$

$$\widetilde{\omega}_\pm(r,v)=\omega+\frac{1}{2\tau_1}\int\frac{d\Omega'}{4\pi}g_\pm(r,v)+\frac{3}{2\tau_{so}}\int\frac{d\Omega'}{4\pi}g_\mp(r,v')\sin^2(\theta-\theta'),$$

$$\widetilde{\Delta}_\pm(\omega,r,v)=\Delta+\frac{1}{2\tau_1}\int\frac{d\Omega'}{4\pi}f_\pm(r,v)+\frac{3}{2\tau_{so}}\int\frac{d\Omega'}{4\pi}f_\mp(r,v')\sin^2(\theta-\theta'), \tag{14}$$

where

$$\tau_1^{-1}=nN(0)|U_1|^2,$$

$$\tau_{so}^{-1}=\frac{2}{3}nN(0)|U_{so}|^2,$$

$$\Delta=\frac{\lambda}{2}\sum_\omega\int\frac{d\Omega}{4\pi}[f_+(\omega,r,v)+f_-(\omega,r,v)].$$

Equations (14) are the generalization of the Eilenberger equations for the case when an exchange field and spin-orbit scattering are present. A heuristic way to obtain these equations would be to generalize the results from Maki[11] and Likharev.[13]

One can see from (14) that in the absence of the spin-orbit scattering the plus and minus components do not mix with each other. It is only τ_{so} that mixes the time reversed states.

III. USADEL EQUATIONS CLOSE TO T_c

The Eilenberger equations can be considerably simplified when the mean free path for potential scattering $l=v_F\tau_1$ is much shorter than the superconducting coherence length. This simplification appears because all the Green's functions corresponding to the different directions of the relative motion of the electron (different $\mathbf{n}$'s) get smeared out on the distances of the order of l, and since the characteristic scale of the Green's functions variations is ξ we obtain that the spherical harmonics expansion is rapidly converging so that we can restrict ourselves to only the first two harmonics.

We will not derive the general case of the Usadel equations in the presence of the spin-orbit scattering but will only restrict ourselves to the temperatures very close to T_c. This case is much simpler for investigation and contains all the new physics of introduction of the spin orbit scattering. For temperatures sufficiently close to T_c we can take $g_\pm\equiv1$ and (10) becomes

$$\left(\widetilde{\omega}_\pm\pm ih-\frac{1}{2}\mathbf{v}\frac{\partial}{\partial\mathbf{R}}\right)f_\pm(\omega,r,v)=\widetilde{\Delta}_\pm(\omega,r,v), \tag{15}$$

$$\left(\widetilde{\omega}_\pm\pm ih+\frac{1}{2}\mathbf{v}\frac{\partial}{\partial\mathbf{R}}\right)f_\pm^\dagger(\omega,r,v)=\widetilde{\Delta}_\pm^*(\omega,r,v), \tag{16}$$

$$\tilde{\omega}_{\pm}(\omega,r,v)=\omega+\frac{1}{2\tau_1}+\frac{1}{\tau_{so}},$$

$$\tilde{\Delta}_{\pm}(\omega,r,v)=\Delta+\frac{1}{2\tau_1}\int\frac{d\Omega'}{4\pi}f_{\pm}(\omega,r,v')+\frac{3}{2\tau_{so}}\int d\,\frac{\Omega'}{4\pi}f_{\mp}(\omega,r,v')\sin^2(\theta-\theta'). \tag{17}$$

We solve Eqs. (15)–(17) for the case when the parameters vary only as a function of x. This corresponds to the ''sandwich'' geometry of interest here.

Let θ be the angle between $\mathbf{n}$ and x axis. From the full Legendre polynomial expansion of $f_{\pm}(\omega,\tau,v)=\Sigma_{l=0}^{\infty}f^{l}(\omega,x)P_l(\cos\theta)$ by the reasons described earlier we take only the first two terms

$$f_{\pm}(\omega,x,v)=f_{\pm}^{0}(\omega,x)+f_{\pm}^{1}(\omega,x)\cos\theta. \tag{18}$$

We integrate Eq. (15) over all θ's directly and after being multiplied by $\cos\theta$ to arrive at the following equations:

$$\left(\omega\pm ih+\frac{1}{2\tau_1}+\frac{1}{\tau_{so}}\right)f_{\pm}^{0}(\omega,x)-\frac{v_F}{6}\frac{\partial}{\partial x}f_{\pm}^{1}(\omega,x)=\Delta+\frac{1}{2\tau_1}f_{\pm}^{0}(\omega,x)+\frac{1}{\tau_{so}}f_{\pm}^{0}(\omega,x), \tag{19}$$

$$\left(\omega\pm ih+\frac{1}{2\tau_1}+\frac{1}{\tau_{so}}\right)f_{\pm}^{1}(\omega,x)-\frac{v_F}{2}\frac{\partial}{\partial x}f_{\pm}^{0}(\omega,x)=0. \tag{20}$$

We are in the limit of a very dirty superconductor, when $1/\tau_1\gg T_c,\ h,\ 1/\tau_{so}$, so Eq. (20) simplifies to

$$f_{\pm}^{1}(\omega,x)=\tau_1 v_F\frac{\partial}{\partial x}f_{\pm}^{0}(\omega,x). \tag{21}$$

Inserting into (19) we get

$$\frac{1}{4}D\frac{\partial^2}{\partial x^2}f_{\pm}^{0}(\omega,x)-(\omega\pm ih)f_{\pm}^{0}+\Delta=\frac{1}{\tau_{so}}[f_{\pm}^{0}(\omega,x)-f_{\mp}^{0}(\omega,x)],$$

$$\Delta(x)=\frac{\lambda}{2}\sum_{\omega}[f_{+}^{0}(\omega,x)+f_{-}^{0}(\omega,x)], \tag{22}$$

where $D=\frac{2}{3}\tau_1 v_F^2$. In the absence of the spin orbit scattering we recover the equations of Ref. 3.

We now want to check the self-consistency of our approximation, namely that the higher harmonics in the Legendre polynomial expansion are small. From Eq. (21) we see that $f_1/f_0\propto l/L$ where L is the characteristic scale on which $f_{\pm}$ changes. Analogously, we can get that $f(n+1)/f(n)\propto l/L$ by multiplying Eq. (15) by $\cos^n\theta$ and integrating over all angles. In the superconductor $L\propto\xi_0$ and in the ferromagnet $L\propto\sqrt{D/h}$ and in both cases $l/L\ll 1$, so that our approximation is valid.

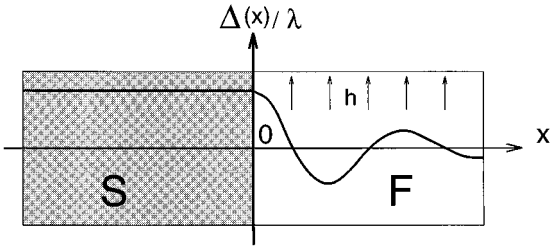

FIG. 3. Superconducting order parameter on the SF interface.

IV. BOUNDARY CONDITIONS FOR THE EILENBERGER AND USADEL EQUATIONS

The boundary conditions for the Eilenberger equations follow from the continuity conditions for the normal and anomalous Greens functions. Ivanov *et al.* showed[14] that at the sharp planar interface the Eilenberger functions are continuous along the flight trajectories on which the electrons can pass from one metal to the other

$$f(x=0-,\mathbf{n})=f(x=0+,\mathbf{n}) \tag{23}$$

and are equal on trajectories corresponding to the incident and reflected waves

$$f(x=0,\mathbf{n}_{+})=f(x=0,\mathbf{n}_{-}). \tag{24}$$

In Eq. (23) we assumed that $x=0$ is the boundary between two metals and in Eq. (24) $x=0$ is a perfectly reflecting boundary.

The Usadel functions are the isotropic part of the Eilenberger functions and obviously they should also satisfy the continuity condition on any boundary. Another condition on these functions comes from the requirement of the continuity of the current and reduces to the conservation of the quantity $DN\nabla F$ where N is the density of states and D is the diffusion coefficient.

Interfacial scattering breaks the validity of the semiclassical approximation. Several authors have found effective boundary conditions that include the effect of δ-function scattering.[15,16] However, those do not appear to be applicable for our particular case. In Sec. VII we will describe qualitatively the effect of the interfacial scattering on the Josephson current in the SFS system.

In the sections to follow we assume for simplicity that the Fermi velocities in the two materials are the same. Whatever difference there may be in these velocities will lead to reflections at the interface, and can also be modeled by a δ-function potential, as we will do in the subsequent article. For now we will justify our assumption by noticing that there is little difference between the Fermi velocities of most conducting ferromagnets and superconductors, leading to a negligible reflection at the interface.

V. PROXIMITY EFFECT IN A FERROMAGNET

As described in the introduction an exchange field in the ferromagnet leads to the oscillations of the induced superconducting order parameter (see Fig. 3). In this section we show that the presence of spin-orbit scattering not only modifies the oscillation length but also leads to an extra de-

cay of the order parameter and, that a critical strength of the spin orbit scattering completely suppresses the oscillations. Clearly the role of the spin-orbit scattering needs to be addressed in interpreting the experimental results on the S-F-S Josephson junctions and S-F bilayers.

We restrict ourselves to the case of dirty superconductors at temperatures close to T_c, when Eqs. (22) are valid. In our model the superconductor is characterized by Δ which we take constant everywhere in the superconductor, which has no exchange field and no spin orbit scattering. In the ferromagnet, the BCS coupling is identically zero, so that $\Delta_F = 0$. However, the induced order parameter $\langle \psi_\downarrow(x)\psi_\uparrow(x)\rangle$ is finite.

The Usadel equations in the superconductor are given by

$$\frac{1}{4} D \frac{\partial^2}{\partial x^2} f_\pm(\omega,x) - \omega f_\pm + \Delta = 0 \tag{25}$$

and in the ferromagnet

$$\frac{1}{4} D \frac{\partial^2}{\partial x^2} f_\pm(\omega,x) - (\omega \pm ih) f_\pm = \frac{1}{\tau_{so}} [f_\pm(\omega,x) - f_\mp(\omega,x)]. \tag{26}$$

We need to solve these equations with the boundary conditions

$$f_\pm^{(F)}|_{x=0+} = f_\pm^{(S)}|_{x=0-},$$

$$\sigma_n \frac{\partial}{\partial x} f_\pm^{(F)}|_{x=0+} = \sigma_s \frac{\partial}{\partial x} f_\pm^{(S)}|_{x=0-}, \tag{27}$$

which correspond to continuity of the order parameter and the current. Another obvious condition is that at $x=-\infty$, in the bulk of the superconductor, the $f_\pm$ functions approach their equilibrium values of Δ/ω.

Since $h \gg T_c$ in most cases of interest, in Eq. (26) we can omit the ω term. As it turns out this way we are only losing the usual decay of the induced order parameter in the normal metal at the distances $\xi_n = \sqrt{D/\Delta}$, because we are interested in the effects of the exchange field, not in the conventional proximity decays. We look for the solution of Eq. (26) in the form

$$\begin{pmatrix} f_+(x) \\ f_-(x) \end{pmatrix} = \begin{pmatrix} C_+ \\ C_- \end{pmatrix} e^{kx} \tag{28}$$

and after substitution into (26), we get the eigenvalue equation for k

$$\left(\frac{1}{4} Dk^2 - \frac{1}{\tau_{so}}\right)^2 + h^2 - \frac{1}{\tau_{so}^2} = 0. \tag{29}$$

This equation has four solutions $\pm k_M$ and $\pm k_M^*$, where

$$k_M^2 = +i \frac{4}{D} \sqrt{h^2 - 1/\tau_{so}^2} + \frac{4}{D\tau_{so}}, \tag{30}$$

$$k_M^{*2} = -i \frac{4}{D} \sqrt{h^2 - 1/\tau_{SO}^2} + \frac{4}{D\tau_{SO}}.$$

The imaginary part of k_M defines the oscillations. We can see from (30) that spin orbit scattering modifies the characteristic length of the oscillations and completely destroys them for $1/\tau_{so} > h$. The analogous expression for the case of strong spin orbit scattering has been obtained by Ref. 17.

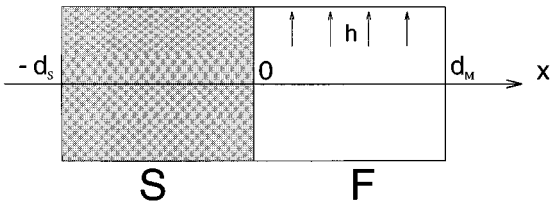

FIG. 4. Geometry of an SF bilayer.

For future reference we find a complete solution of the Usadel equations in the ferromagnet and the superconductor, which means that we need to find the eigenvectors corresponding to each eigenvalue.

For the ferromagnet

$$\pm k = k_M = \left(\frac{4}{D\tau_{so}} + \frac{4i}{D} \sqrt{h^2 - 1/\tau_{so}^2}\right)^{1/2},$$

$$\frac{C_+}{C_-} = \frac{i/\tau_{so}}{\sqrt{h^2 - 1/\tau_{so}^2} - h}, \tag{31}$$

$$\pm k = k_M^* = \left(\frac{4}{D\tau_{so}} - \frac{4i}{D} \sqrt{h^2 - 1/\tau_{so}^2}\right)^{1/2},$$

$$\frac{C_+}{C_-} = -\frac{\sqrt{h^2 - 1/\tau_{so}^2} - h}{i/\tau_{so}}. \tag{32}$$

So, the general form of the solution in the ferromagnet is

$$\begin{pmatrix} f_+(\omega,x) \\ f_-(\omega,x) \end{pmatrix} = C_1 \begin{pmatrix} i\alpha \\ 1 \end{pmatrix} e^{k_M x} + C_2 \begin{pmatrix} i\alpha \\ 1 \end{pmatrix} e^{-k_M x} + C_3 \begin{pmatrix} 1 \\ -i\alpha \end{pmatrix} e^{k_M^* x} + C_4 \begin{pmatrix} 1 \\ -i\alpha \end{pmatrix} e^{-k_M^* x}, \tag{33}$$

where

$$\alpha = \frac{1/\tau_{so}}{\sqrt{h^2 - 1/\tau_{so}^2} - h}. \tag{34}$$

And for the superconductor

$$\pm k = k_S = \sqrt{4\omega/D} \tag{35}$$

and the solution is

$$\begin{pmatrix} f_+(\omega,x) \\ f_-(\omega,x) \end{pmatrix} = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} e^{k_S x} + \begin{pmatrix} C_3 \\ C_4 \end{pmatrix} e^{-k_S x}. \tag{36}$$

VI. T_c FOR AN S-F BILAYER

In this section we apply the general equations developed above to determine the superconducting transition temperature of the superconductor-ferromagnet bilayer (see Fig. 4 for geometry). The difference between our consideration and that by Ref. 2 is the inclusion of the spin-orbit scattering.

For this problem it is more convenient to work with

$$F_+(\omega,x) = \frac{1}{2} [f_+(\omega,x) + f_-(\omega,x)],$$

$$F_-(\omega,x)=f_+(\omega,x)-f_-(\omega,x), \tag{37}$$

rather than with $f_\pm$. We have the following equations for F's.

In the superconductor, $x<0$:

$$D\frac{\partial^2}{\partial x^2}F_+(\omega,x)-|\omega|F_+(\omega,x)+\Delta=0,$$

$$D\frac{\partial^2}{\partial x^2}F_-(\omega,x)-|\omega|F_-(\omega,x)=0, \tag{38}$$

and we have a self-consistency equation $\Delta(x)=\lambda\Sigma_\omega F_+(\omega,x)$.

In the ferromagnet, $x>0$:

$$D\frac{\partial^2}{\partial x^2}F_+(\omega,x)-\frac{ih}{2}F_-(\omega,x)=0,$$

$$D\frac{\partial^2}{\partial x^2}F_-(\omega,x)-\frac{ih}{2}F_+(\omega,x)=\frac{2}{\tau_{so}}F_-(\omega,x). \tag{39}$$

And in the last two equations we also neglected ω in comparison with h which means that the thickness of the ferromagnetic layer should be smaller than $\sqrt{D/\Delta}$.

Let us take solutions of F_+ in the superconductor of the form

$$F_+(\omega,x)=C_+(\omega)\cos[k_s(x+d_s)]. \tag{40}$$

This will satisfy Eq. (38) if we take

$$C_+(\omega)=\frac{\Delta}{Dk_s^2+|\omega|}. \tag{41}$$

For F_- we take

$$F_-(\omega,x)=C_-(\omega)\cos[\sqrt{|\omega|/D}(x+d_s)]. \tag{42}$$

Both (40) and (42) satisfy the condition $F'_\pm(x=-d_s)=0$ (no current) on the left boundary of the superconductor.

The self-consistency equation in the superconductor is now given by

$$\lambda\sum_\omega\frac{1}{Dk_s^2+|\omega|}=1. \tag{43}$$

After finding k_s from the boundary conditions, equation (43) gives the transition temperature.

In the ferromagnet, the solution of (39) that satisfies the boundary condition at $x=d_m$ is

$$\begin{pmatrix}F_+(\omega,x)\\F_-(\omega,x)\end{pmatrix}=C_{1F}\begin{pmatrix}\frac{ih}{2Dk_M^2}\\1\end{pmatrix}\cosh[k_M(x-d_M)]+C_{2F}\begin{pmatrix}\frac{ih}{2Dk_M^{*2}}\\1\end{pmatrix}\cosh[k_M^*(x-d_M)]. \tag{44}$$

Introducing $\kappa=C_{2F}/C_{1F}$, the boundary condition at the S-F interface $x=0$ becomes a set of two equations on κ and k_s (since the equations are linear it is sufficient to require the continuity of $\sigma F'$-F at the interface)

$$\sigma_s\sqrt{|\omega|/D}\tanh[\sqrt{|\omega|/D}d_s]=-\sigma_n\frac{k_M\sinh(k_Md_M)+\kappa\left(\frac{k_M^*}{k_M}\right)^2k_M^*\sinh(k_M^*d_M)}{\cosh(k_Md_M)+\kappa\left(\frac{k_M^*}{k_M}\right)^2k_M^*\cosh(k_M^*d_M)},$$

$$\sigma_sk_s\tan(k_sd_s)=\sigma_n\frac{k_M\sinh(k_Md_M)+\kappa k_M^*\sinh(k_M^*d_M)}{\cosh(k_Md_M)+\kappa k_M^*\cosh(k_M^*d_M)}. \tag{45}$$

In order for our assumption of the separation of ω and x dependences in F_+ to be valid we must have k_s and correspondingly κ independent of ω. As can be easily seen from (45) this is only true when $\sigma_sk_s\gg\sigma_Fk_M$ or $\sigma_sk_s\ll\sigma_Fk_M$. We will consider the first case, and the second one can be done in a similar fashion.

So, for $\sigma_sk_s\gg\sigma_Fk_M$,

$$\kappa=-\left(\frac{k_M}{k_M^*}\right)^2\frac{\cosh(k_Md_M)}{\cosh(k_M^*d_M)} \tag{46}$$

and for k_s we have

$$k_s\tan(k_sd_s)=\frac{\sigma_n}{\sigma_s}\frac{k_M^*\tan(k_Md_M)-k_M\tan(k_M^*d_M)}{\frac{k_M^*}{k_M}-\frac{k_M}{k_M^*}}=\frac{\sigma_n}{\sigma_s}\frac{-i\,\mathrm{Im}k_M}{\frac{k_M^*}{k_M}-\frac{k_M}{k_M^*}}\frac{\sinh(2\,\mathrm{Re}k_Md_M)}{\cosh(k_Md_M)\cosh(k_M^*d_M)}\times\left[1-\frac{\mathrm{Re}k_M}{\mathrm{Im}k_M}\frac{\sinh(2\,\mathrm{Im}k_Md_M)}{\sinh(2\,\mathrm{Re}k_Md_M)}\right]. \tag{47}$$

If we introduce θ,

$$\tan2\theta=\frac{\sqrt{h^2-1/\tau_{so}^2}}{1/\tau_{so}}, \tag{48}$$

we can write $k_M=|k_M|e^{i\theta}$. And if now we assume $|k_M|d_M\cos\theta\gg1$ and take into account $\sigma_sk_s\gg\sigma_Fk_M$ expression (47) simplifies to

$$k_s^2=\frac{\sigma_n}{\sigma_s}\frac{|k_M|}{2d_s\cos\theta}[1-2e^{-2|k_M|\cos\theta d_M}\sin(2|k_M|\sin\theta)]. \tag{49}$$

The transition temperature for the bilayer may be found from

$$\rho_c=\frac{Dk_s^2}{2\pi T_C},$$

$$\ln\frac{T_{c0}}{T_c}=\psi\left(\frac{1}{2}+\rho_c\right)-\psi\left(\frac{1}{2}\right). \tag{50}$$

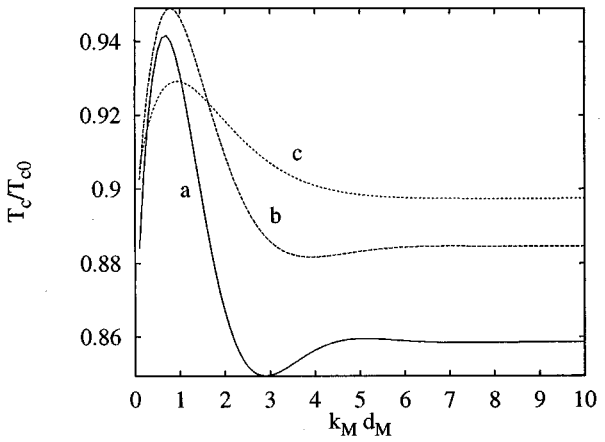

FIG. 5. Transition temperature of an SF bilayer as a function of the magnetic layer thickness for different values of the spin orbit scattering. Curve **a** is for $1/\tau_{so}=0$, curve **b** for $1/\tau_{so}=0.5$ h, and curve **c** for $1/\tau_{so}=0.9$ h. Other parameters were taken to satisfy $\pi D\sigma_n|k_M|/4d_s\sigma_s T_{c0}=0.1$.

In Fig. 5 we show how the oscillations of T_c are modified by the presence of the spin orbit scattering.

We can see that the oscillations of T_c as a function of the ferromagnetic layer thickness found by Ref. 2 are considerably modified by the presence of the spin orbit scattering.

VII. JOSEPHSON EFFECT IN THE SFS SANDWICH

We assume that the phase of the left superconductor is $-\phi/2$ and the phase of the right one is $\phi/2$. Fermi velocity, electron masses, and all other parameters are taken equal for both superconductors and the ferromagnet (see Fig. 6 for geometry).

As we have shown earlier, in the presence of the spin-orbit scattering f_+ and f_- are not eigenfunctions of the Usadel equations. It is more convenient therefore to use the basis

$$\hat{e}_1=\begin{pmatrix} i\alpha \\ 1 \end{pmatrix}, \quad \hat{e}_2=\begin{pmatrix} 1 \\ -i\alpha \end{pmatrix} \tag{51}$$

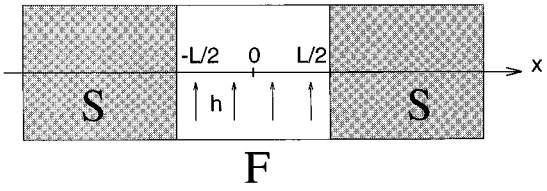

FIG. 6. Geometry of the SFS trilayer junction.

that we have derived in Sec. V. In the bulk of the superconductor

$$\begin{pmatrix} f_+ \\ f_- \end{pmatrix}=\frac{\Delta}{\omega}\begin{pmatrix} 1 \\ 1 \end{pmatrix}=\frac{1+i\alpha}{1-\alpha^2}\frac{\Delta}{\omega}\hat{e}_1+\frac{1-i\alpha}{1-\alpha^2}\frac{\Delta}{\omega}\hat{e}_2 \tag{52}$$

and now we can solve for $\hat{e}_1$ and $\hat{e}_2$ components separately. Matching the values of the functions and the derivatives on the boundaries we get after simple calculations that the Josephson current near T_c is given by

$$I_s=2\pi N(0)DT_c\sum_\omega \frac{\Delta^2}{\omega^2}\left\{\frac{k_M}{\sinh(k_M L)}+\frac{k_M^*}{\sinh(k_M^* L)}\right.$$
$$\left.+\frac{2i\alpha}{1-\alpha^2}\left[\frac{k_M}{\sinh(k_M L)}-\frac{k_M^*}{\sinh(k_M^* L)}\right]\right\}. \tag{53}$$

We can again see that the spin orbit scattering considerably modifies the answer.

Interfacial scattering at the two interfaces modifies this result. Using Bogolyubov–de Gennes equations as in Ref. 18 one can show that in the lowest order in transmission coefficients the right-hand side of Eq. (53) is multiplied by the product of the transmission coefficients, which is an intuitively clear result. We plan to present the details of these calculations elsewhere.

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*On leave of absence from the Department of Physics, University of Notre Dame, Notre Dame, Indiana 46556.

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